# Request for Renewal of Math 135 as an FS Course

Kapiolani Community College, Fall 2013

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I. Course Description

MATH 135 Elementary Functions

3 hours lecture per week

Prerequisite(s): MATH 103 with a grade of “C” or higher or qualification on a math placement test for MATH 135.

MATH 135 focuses elementary functions and graphs, polynomials, absolute values, inequalities, logarithms and exponentials.

Upon successful completion of MATH 135 the student should be able to:

1. Apply definitions of functions, inverse functions, and composite functions.

2. Show familiarity with all principles involving linear functions.

3. Find roots, evaluate, sketch, and solve inequalities involving polynomial functions.

4. Graph rational functions using the concepts of asymptotes.

5. Apply definitions and principles of logarithmic and exponential functions.

6. Use knowledge and techniques of this course in solving applied problems.
II. Changes

No significant changes have been made in Math 135 since the last request for foundation designation was approved.

III. Assessing the Course

Hallmark 1. Expose students to the beauty, power, clarity and precision of formal systems.

Mathematics is probably the most prevalent and powerful formal system humans use. In Elementary Functions, students are imbued with the notation and application of the formal system of functions of real numbers. Complicated ideas can be captured elegantly with this system. For example, an important application of a topic from MATH 135 is using the difference quotient to capture average rates of change. Classical physics would be impossible without this simple notation, as shown here in various forms:

\[ \frac{f(x + h) - f(x)}{h} \]

For \( f(x) = \sqrt{x} + 1 \), evaluate and simplify the difference quotient \( \frac{f(x + h) - f(x)}{h} \).

A change of \( h \) in the independent variable \( x \) of this radical function produces a change in the function value shown as this difference quotient. With the power of mathematical notation, this expression can now be used to quickly calculate the change in the function for any nonzero value of \( h \). This has applications to motion, velocity (the change is the average velocity over an interval) and economics (if \( f \) is a cost function it is related to marginal cost.) Of course, knowledge of the rules of algebra is required to work with this notation, and MATH 135 students are constantly reinforcing their skills in algebra, perhaps the most ancient and powerful human formal system. In addition, the difference quotient is crucial to the differential calculus, so MATH 135 students are preparing for calculus, a great mathematical accomplishment.

Hallmark 2. Help students understand the concept of proof as a chain of inferences

Proofs of many of the results are given in this class, and students are required to
show steps in various exercises using chains of inferences. For example, the students learn the formal definition of a one-to-one function, and are required to prove that certain functions are one-to-one.

**Definition:** A function \( f \) is one-to-one if for all \( x_1, x_2 \) in the domain of \( f \), \( f(x_1) = f(x_2) \) implies \( x_1 = x_2 \).

**Exercise:** Show that the function \( f(x) = \frac{x - 5}{x - 1} \) is one-to-one.

**Solution:** Suppose that \( f(x_1) = f(x_2) \). Then
\[
\begin{align*}
\frac{x_1 - 5}{x_1 - 1} &= \frac{x_2 - 5}{x_2 - 1} \\
(x_1 - 5)(x_2 - 1) &= (x_2 - 5)(x_1 - 1) \\
x_1x_2 - x_1 - 5x_2 + 5 &= x_2x_1 - 5x_1 - x_2 + 5 \\
-x_1 + 5x_2 &= -5x_1 - x_2 \\
4x_1 &= 4x_2 \\
x_1 &= x_2
\end{align*}
\]

Hence \( f \) is one-to-one.

The display follows a logical progression dictated by the rules of algebra, building from simple steps a more complicated result. Such is an example of a chain of inferences, and the students practice these arguments.

**Hallmark 3: Teach students how to apply formal rules or algorithms.**

An important aspect of MATH 135 is learning synthetic division and other algorithms to find roots of polynomials or simplify a rational expression. The process of synthetic division is shown here:

**Exercise:** Use synthetic division and the Remainder Theorem to evaluate the polynomial \( f(x) = x^4 - x^3 + 7x^2 - 8x + 1 \) at \( x = 2 \).

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Solution

By the Remainder Theorem \( f(2) = 21 \).

The algorithm applied can be simpler than evaluating the polynomial by direct
substitution, and the Remainder Theorem shows students the power of algorithms. Students also use synthetic division to factor polynomials functions to sketch graphs. They are exposed to Descartes' Rule of Signs, the Rational Zeros Theorem, and techniques to find bounds on the roots of a polynomial when the roots can only be approximated without more advanced methods. The combination of algorithms and rules gives a complicated mix that can give students deep insight into famously difficult problems.

**Hallmark 4: Require students to use appropriate symbolic techniques in the context of problem solving, and in the presentation and critical evaluation of evidence.**

The key to applied mathematics is careful description of the variables and use of symbolic techniques to solve problems. Students in MATH 135 are shown the appropriate notation and layout in using mathematics to answer a question in which the quantities involved are well-defined. Problems involving direct and inverse variation give examples from concrete science that students use mathematics to solve.

*Exercise:* If gravitational force between two bodies varies inversely as the distance between the bodies, and the force is 100N when the distance is 100km, find the force when the distance is 200km.

*Solution:* Let $F$ denote the force between the bodies and $d$ the distance between the bodies. The inverse variation of the two quantities implies that

$$F = \frac{k}{d^2},$$

for some fixed constant $k$. Since a distance of 100km gives a force of 100N, we can find the constant $k$:

$$100 = \frac{k}{100^2} \Rightarrow k = 10^6.$$  

The force at a distance of 200km is therefore

$$F = \frac{10^6}{200^2} = 25,$$

i.e., the force is 25N.

Other examples in which students practice using symbolic techniques in the context of problem solving arise in the material on logarithmic and exponential functions. The wide variety of applications gives students a wealth of experience with applied problem solving in such areas as health (the processing of a drug by the human body proceeds according to an exponential function), finance (compound interest is explained and students are shown how to calculate interest growth and amortization of a loan), and physics (Newton’s
Law of Cooling is an example of applied exponential symbolic techniques.

**Hallmark 5: Not focus solely on computational skills.**

A variety of skills, not solely computational, are involved in MATH 135. There are many examples of interpretative, creative and spatial/visual/artistic skills required. A major goal of the class is to teach students graphing techniques for the functions studied. The graphing techniques require knowledge of symmetry and distance, and creating the graphs demands drawing ability. In fact, mathematics and art have always shared intimate connections. Below are examples of routine graphs that the students will produce in MATH 135. They are trained to note symmetry and use it to aid the drawing.

![Graph](image)

**Figure 1:** The graph of $y = \frac{x}{x^2 - 1}$. 

Figure 2: The graph of \( y = 2x^2 - x^4 \).

Figure 3: The graphs of \( y = e^x \) and \( y = \ln x \).
In fact, the power of the Cartesian coordinate system is that it goes beyond computation, taking algebraic equations and making them visual, which allows deep insight and easy analysis.

**Hallmark 6: Build a bridge from theory to practice and show the students how to traverse that bridge.**

As mathematics itself is a practice, students are on the road to being a mathematician. Although the road is long and students are not expected to become mathematicians, every mathematician has mastered the skills in MATH 135. Mathematics exercises are the first step in learning how to do mathematical research.

There are many applications given in MATH 135. In MATH 135, the expectation is that the mathematics will be practiced in the context of a science. Below are several examples of applied problems shown to the students in MATH 135.

1. **First-Class Diver** The pressure exerted by water at a point below the surface varies directly with the depth. The pressure is $4.34 \text{ lb/in.}^2$ at a depth of 10 ft. What pressure does the sperm whale (the deepest diver among the air-breathing mammals) experience when it dives 6000 ft below the surface?

2. Jang’s Postal Service charges $35 for addressing 200 envelopes, and $60 for addressing 400 envelopes. What is the average rate of change of the cost as the number of envelopes goes from 200 to 400?

3. **Admission to the Zoo** Winona paid $100 for a lifetime membership to Friends of the Zoo, so that she could gain admittance to the zoo for only $1 per visit. Write Winona’s average cost per visit $C$ as a function of the number of visits when she has visited $x$ times. What is her average cost per visit when she has visited the zoo 100 times? Graph the function for $x > 0$. What happens to her average cost per visit if she starts when she is young and visits the zoo every day?
4.  

*Future Value*  If $30,000 is deposited in First American Savings and Loan in an account paying 6.18% compounded continuously, then what will be the value of the account after 12 years and 3 months?

5.  

*Time of Death*  A detective discovered a body in a vacant lot at 7 A.M. and found that the body temperature was 80°F. The county coroner examined the body at 8 A.M. and found that the body temperature was 72°. Assuming that the body temperature was 98° when the person died and that the air temperature was a constant 40° all night, what was the approximate time of death?